ANSWER KEY PD 1

CIRCULAR MOTION

&

GRAVITY

AT CIRCULAR MOTION GRAVITY (17)

Directions: Solve the following problem. Show all work. Be neat. Your solution should mathematically read like an essay. Each question is worth 5 points

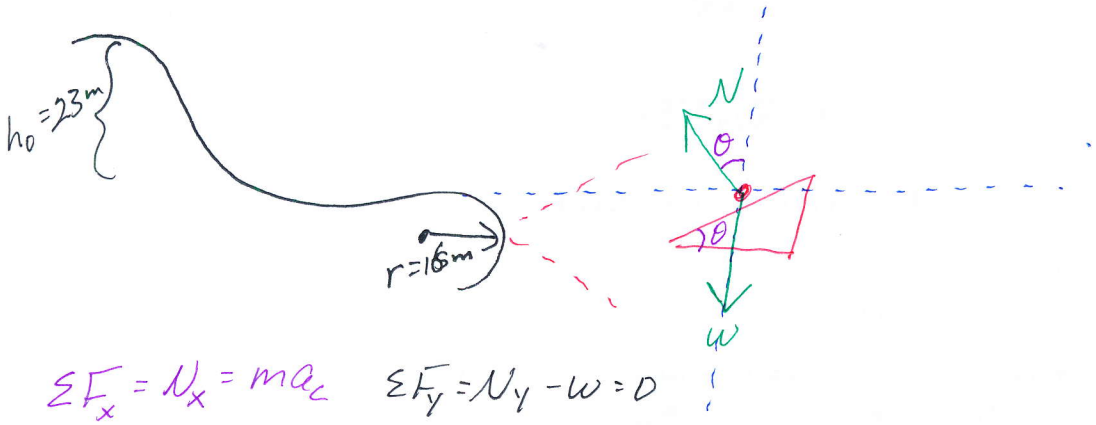
1) An analog clock is made of an hour hand, a minute hand, and a second hand. Determine the period of the minute hand, measured in seconds.

$\left(\frac{1\,Rev}{1\,hr}\right)$(... No ...)

$\left(\frac{1\,hr}{1\,Rev}\right)\left(\frac{60\,min}{1\,hr}\right)\left(\frac{60\,sec}{1\,min}\right) = 3600\,s$

Key $G=6.67\times10^{-11}\,Nm^2/kg^2$ $m_{earth}=5.98\times10^{24}kg$ $r_{earth}=6.37\times10^{6}m$ $m_{moon}=7.35\times10^{22}kg$ $r_{moon}=1.74\times10^{6}m$

2) A roller coaster is initially at a height of 23 m above the ground. The track drops at a gentle slope of 30 degrees until the track is at ground level, then progresses through a banked turn of radius 16m. Determine the banking, as measured from the horizontal, so the riders on the train feel no frictional forces between themselves and the seat, and no forces acting from the sides of the seat. (Only force acting between the rider and the seat is the normal from the bottom of the seat.)



$\Sigma F_x = N_x = ma_c$ $\quad \Sigma F_y = N_y - w = 0$

$N\sin\theta = \frac{mv^2}{r}$ $\quad N\cos\theta = mg$

$N = \frac{mg}{\cos\theta}$

$\frac{mg}{\cos\theta}\sin\theta = \frac{mv^2}{r}$

$g\tan\theta = \frac{v^2}{r}$

$PE_0 = KE$

$mgh_0 = \frac{1}{2}mv^2$

$2gh_0 = v^2$

$g\tan\theta = \frac{2gh_0}{r}$

$\tan\theta = \frac{2h_0}{r}$

$\theta = \tan^{-1}\left[\frac{2h_0}{r}\right]$

$\theta = \tan^{-1}\left[\frac{2(23m)}{16m}\right] = 70^\circ$

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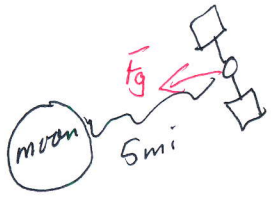
3) Determine the period of orbit for a spacecraft that orbits the moon 5 miles above the surface of the moon. Assume a circular orbit.



$\Sigma F_{\perp} = F_g = ma_c$

$\frac{Gmm}{r^2} = \frac{mv^2}{r}$

$\frac{Gm}{r} = v^2$

$\frac{Gm}{r} = \frac{4\pi^2 r^2}{T^2}$

$T = \sqrt{\frac{4\pi^2 r^3}{Gm_m}}$

$(5mi)\left(\frac{1609m}{1mi}\right) = 8045m$

$1.74\times10^6 m + 8045mi$

$1.748\times10^6 m$

$T = \sqrt{\frac{4\pi^2(1.748\times10^6 m)^3}{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(7.35\times10^{22}kg)}} = 6.55\times10^3 s = 109 min$

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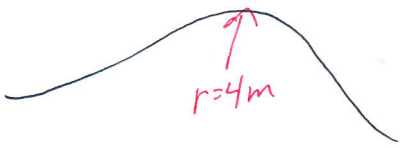
4) A particular road follows the terrain, which goes over a hill. The radius of curvature at the top of the hill is 4m. Determine the greatest speed that a vehicle can have going over the hill without going airborne. Assume the mass of the car is 2,000 kg.



Free Body

$\Sigma F_y = W = ma_c$

$mg = ma_c$

$g = a_c$

$g = \frac{v^2}{r}$

$\sqrt{gr} = v$

$\sqrt{(9.8 m/s^2)(4m)} = 6.3 m/s$

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5) A wheel of radius 0.32 m is rotating at 4.6 rev/s. Determine the linear speed of the wheel.

$v = 2\pi r f$

$v = 2\pi(.32\,m)(4.6\,rev/s)$

$v = 9.2\,m/s$

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6) A wheel of radius 0.32 m is rotating at 4.6 rev/s. Determine the angular speed of the wheel.

$\omega = 2\pi f$

$\omega = 2\pi(4.6\ \text{Rev/s})$

$\omega = 28.8\ \text{rad/s}$

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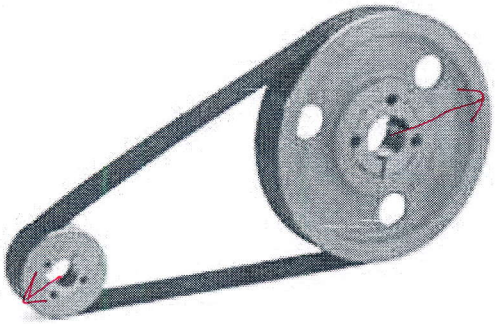
7) The pulley radii are 12 cm and 2 cm. If the belt runs at a linear speed of 2.3 m/s, determine the frequency of the large and small pulley.



$r = 12cm = .12m$

$r = .02m$

Small

$v = 2\pi rf$

$\frac{v}{2\pi r} = f$

$\frac{2.3 m/s}{2\pi(.02m)} = 18 Rev/s$

Large

$v = 2\pi rf$

$\frac{v}{2\pi r} = f$

$\frac{2.3 m/s}{2\pi(.12m)} = 3.05 Rev/s$

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