

Level I Answer Key Pd 3

Circular Motion

AT Circular Motion (21)

Directions: Solve the following problem. Show all work. Be neat. Your solution should mathematically read like an essay. This is worth 50 points.

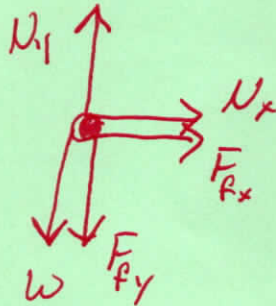
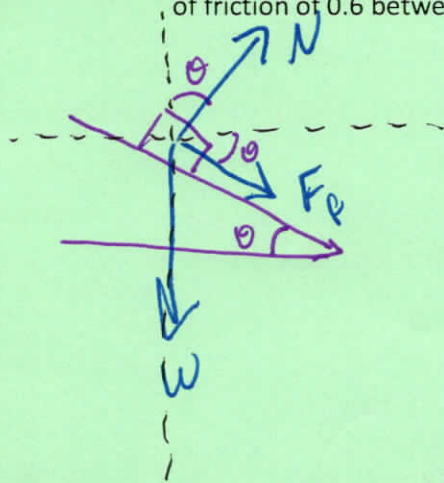
- 1) A rubber stopper of mass 23g is at the end of a 1.2 m long string. The stopper is spun in a vertical circle such that it makes 25 revolutions in 12 seconds. Determine the period of the stopper.



$$\frac{12s}{25 \text{ Rev}} = 48 \text{ s/Rev}$$

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- 2) A modern Honda accord has a mass of about 1,400 kg. Determine the greatest speed that an Accord can go through a turn of radius 35m that is banked at 15 degrees. Assume a coefficient of friction of 0.6 between the tires and the pavement.



$$\begin{aligned} \Sigma F_x &= N_x + F_{fx} = ma \\ N \sin \theta + F_f \cos \theta &= \frac{mv^2}{r} \\ N \sin \theta + \mu N \cos \theta &= \frac{mv^2}{r} \\ N(\sin \theta + \mu \cos \theta) &= \frac{mv^2}{r} \end{aligned}$$

$$\Sigma F_y = N_y - W - F_{fy} = ma$$

$$N \cos \theta - W - F_f \sin \theta = 0$$

$$N \cos \theta = W + F_f \sin \theta$$

$$\cos \theta N = mg + \mu N \sin \theta \quad (\text{opps})$$

$$N \cos \theta + \mu N \sin \theta = mg$$

$$N(\cos \theta + \mu \sin \theta) = mg$$

$$N = \frac{mg}{\cos \theta + \mu \sin \theta}$$

$$\frac{mg(\sin \theta + \mu \cos \theta)}{(\cos \theta + \mu \sin \theta)} = \frac{mv^2}{r}$$

$$\frac{g(\sin \theta + \mu \cos \theta)}{(\cos \theta + \mu \sin \theta)} = \frac{v^2}{r}$$

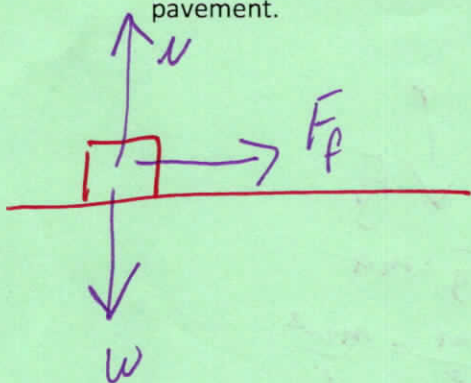
$$\frac{gr(\sin \theta + \mu \cos \theta)}{(\cos \theta + \mu \sin \theta)} = v^2$$

$$v = \frac{mg}{\cos \theta + \mu \sin \theta} \sqrt{\frac{(9.8 \frac{m}{s^2})(35m)(\sin 15 + (0.6)(\cos 15))}{[\cos(15) + (0.6)(\sin 15)]}} = v = \frac{16 \text{ m/s}}{18.53 \text{ m/s}}$$

$$\sqrt{\frac{16}{18.53}}$$

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- 3) Determine the greatest speed that an Accord can go through a turn of radius 35m that is not banked; the turn is flat. Assume a coefficient of friction of 0.6 between the tires and the pavement.



$$\Sigma F_y = N - W = 0$$

$$N = W$$

$$N = mg$$

$$\Sigma F_x = F_f = ma$$

$$\mu N = \frac{mv^2}{r}$$

$$\mu mg = \frac{mv^2}{r}$$

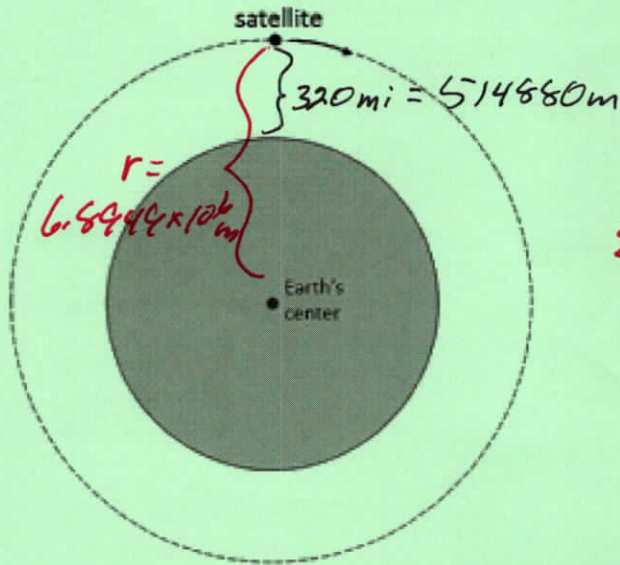
$$\mu g = \frac{v^2}{r}$$

$$\sqrt{\mu gr} = \sqrt{(0.6)(9.8 \text{ m/s}^2)(35 \text{ m})} = 14.3 \text{ m/s}$$

Notice the Similarity / Diff w/ #9

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- 4) Determine how long it will take a satellite to orbit in a circular orbit that is 320 miles above the surface of the Earth.



$$\downarrow F_g$$

$$\Sigma F = F_g = ma$$

$$\frac{Gmm}{r^2} = \frac{mv^2}{r}$$

$$\frac{Gm}{r} = v^2$$

$$\frac{Gm}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{Gm}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{Gm}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{Gm}$$

$$T = \sqrt{\frac{4\pi^2 (6.8949 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) (5.98 \times 10^{24} \text{ kg})}}$$

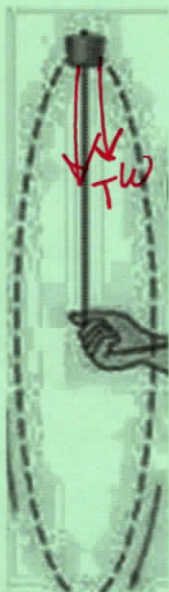
$$T = 5.69 \times 10^3 \text{ s} = 94 \text{ min}$$

$$94.9 \text{ min}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \quad m_{\text{earth}} = 5.98 \times 10^{24} \text{ kg} \quad r_{\text{earth}} = 6.38 \times 10^6 \text{ m} \quad m_{\text{moon}} = 7.35 \times 10^{22} \text{ kg} \quad r_{\text{moon}} = 1.74 \times 10^6 \text{ m}$$

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- 5) A rubber stopper of mass 23g is at the end of a 1.2 m long string. The stopper is spun in a vertical circle such that it makes 25 revolutions in 12 seconds. Determine the tension in the string when the stopper is at its highest point.



$$\Sigma F_y = -T - W = -ma$$

$$T + W = \frac{mv^2}{r}$$

$$T + mg = \frac{m4\pi^2 r^2 f^2}{r}$$

$$T + mg = m4\pi^2 r f^2$$

$$T = m4\pi^2 r f^2 - mg$$

$$T = m(4\pi^2 r f^2 - g) \quad \leftarrow \text{Should make sense...}$$

$$T = (0.0231\text{kg}) \left[4\pi^2 \left(\frac{25\text{rev}}{12\text{s}} \right)^2 (1.2\text{m}) - 9.8\frac{\text{m}}{\text{s}^2} \right] = 4.5\text{N}$$

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- 6) A rubber stopper of mass 23g is at the end of a 1.2 m long string. The stopper is spun in a vertical circle such that it makes 25 revolutions in 12 seconds. Determine the tension in the string when the stopper is at its lowest point.



$$\Sigma F_y = T - W = ma$$

$$T = ma + W$$

$$T = \frac{mv^2}{r} + mg$$

$$T = \frac{m 4\pi^2 r^2 f^2}{r} + mg$$

$$T = m 4\pi^2 r f^2 + mg$$

$$T = m [4\pi^2 r f^2 + g]$$

$$T = (.023 \text{ kg}) \left[(4\pi^2)(1.2 \text{ m}) \left(\frac{25 \text{ Rev}}{12 \text{ s}} \right)^2 + (9.8 \text{ m/s}^2) \right]$$

$$T = 4.95 \text{ N}$$