Answer Key Pd 1

Rotational Dynamics; Day 1

AT Rotational dynamics (18)

Directions: Solve the following problem. Show all work. Be neat. Your solution should mathematically read like an essay.

1) A wheel starts at rest and angularly accelerates at 3 rad/s^2. Determine the angular speed of the wheel after it makes 4 complete rotations.

$\omega_0 = 0$

$\omega = ?$

$\theta = 4(2\pi)$

$\alpha = 3 \text{ rad/s}^2$

$v^2 = v_0^2 + 2ax$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\omega = \sqrt{(2)(3 \text{ rad/s}^2)(8\pi)}$

$\omega = 12.3 \text{ rad/s}$

Answer Key Pd 1

Rotational Inertia

Particle $I = mr^2$

Solid cylinder $I = (1/2)mr^2$

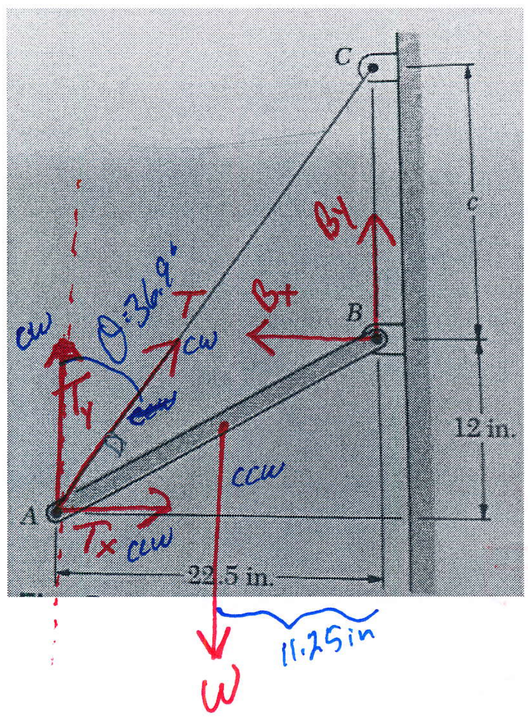
Stick about end $I = (1/3)mL^2$

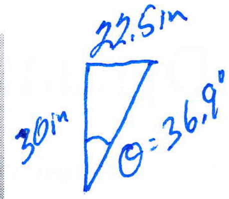
Solid Sphere $I = (2/5)mr^2$

Stick about middle $I = (1/12)mL^2$

2) The bar has a weight of 6.3 lbs. Determine the tension in the string (in pounds) and the reaction at "B" in pounds.

$C = 18 in$





$$\Sigma \tau_B = \tau_{T_y} - \tau_{T_x} - \tau_W = 0$$

$$T_y(22.5in) - T_x(12in) - W(11.25in) = 0$$

$$T(\cos 36.9°)(22.5in) - T\sin(36.9)(12in) - (6.3lb)(11.25in) = 0$$

$$T[\cos(36.9)(22.5in) - \sin(36.9)(12in)] = (6.3lb)(11.25in)$$

$$T = \frac{(6.3lb)(11.25in)}{[\cos(36.9)(22.5in) - \sin(36.9)(12in)]} = \boxed{6.57lb = T}$$

$$\Sigma F_y = B_y + T_y - W = 0$$

$$B_y = W - B_y$$

$$B_y = (6.3lb) - (6.57lb)(\cos 36.9°)$$

$$\boxed{B_y = 1.05lb}$$

$$\Sigma F_x = T_x - B_x = 0$$

$$T_x = B_x$$

$$T(\sin 36.9) = B_x$$

$$(6.57lb)(\sin 36.9) = \boxed{3.94lb}$$

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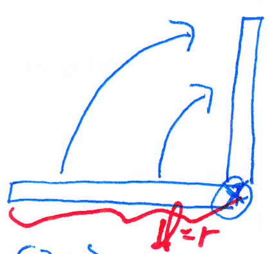
3) A 1.7m long, 1.4kg stick is rotated about an end. With the stick starting at rest, determine the torque that needs to be applied to the stick so that the end of the stick is traveling at 32 m/s once the stick has rotated through a quarter rotation. (Think of situations where this actually happens! There are many!)



$m = 1.4kg$

$\ell = 1.7m$

$\omega_0 = \text{zero}$

$\alpha = ?$

$\tau = I\alpha$

$\omega = \frac{v}{r}$

$I = \frac{1}{3}mL^2$

$\theta = \frac{2\pi}{4} = \frac{\pi}{2}$

$v^2 = v_0^2 + 2ax$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\omega^2 = 2\alpha\theta$

$\omega^2 = \frac{2\tau\theta}{I}$

$\omega^2 = \frac{2\tau\theta}{\frac{1}{3}mL^2}$

$\omega^2 = \frac{2\tau\theta 3}{mL^2}$ $r = L$

$\frac{v^2}{r^2} = \frac{6\tau\theta}{mr^2}$

~~$v^2 = 6\tau\theta$~~

~~$\frac{v^2}{6\theta} = \tau$~~

~~$\frac{(32 m/s)^2}{6(\frac{\pi}{2})} = 108$~~

Opps... I lost mass

$v^2 = \frac{6\tau\theta}{m}$

$\left(\frac{v^2 m}{6(\frac{\pi}{2})}\right) = \tau$

$\frac{v^2 m}{3\pi} = \tau$

$\frac{(32 m/s)^2(1.4kg)}{3\pi} = 152 \frac{kgm^2}{s^2}$

Nm

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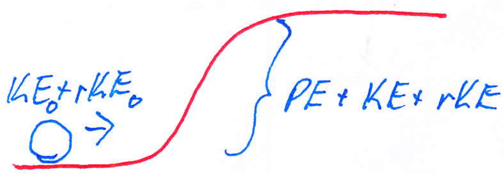
$I = \frac{2}{5}mr^2$

4) A ball is going to be rolled up a hill of height 0.63m. Determine the speed linear speed of the ball required at the bottom of the hill for the ball to top the hill traveling at 0.32 m/s



$$KE_0 + rKE_0 = PE + KE + rKE$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 = mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_0^2}{r^2}\right) = mgh + \frac{1}{2}mv^2 + \left(\frac{1}{2}\right)\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2}$$

$$\frac{1}{2}v_0^2 + \frac{1}{5}v_0^2 = gh + \frac{1}{2}v^2 + \frac{1}{5}v^2$$

$$\frac{7}{10}v_0^2 = gh + \frac{7}{10}v^2$$

$$v_0 = \sqrt{\frac{10}{7}\left[gh + \frac{7}{10}v^2\right]}$$

$$v_0 = \sqrt{\frac{10}{7}\left[\left(9.8\frac{m}{s^2}\right)(.63m) + \left(\frac{7}{10}\right)(.32\,m/s)^2\right]}$$

$$v_0 = 2.99\ m/s$$

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